Kinetic theory of the stability of the inhomogeneous plasmas with shear flows.

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Abstract. The stability of a magnetized inhomogeneous plasma with transverse and field-aligned shear flows is analyzed by using a kinetic formalism. The perpendicular shear flow is shown to be stabilizing, but the field-aligned shear flow is destabilizing. New types of the ion cyclotron and drift-type instabilities are found. A renormalized nonlinear dispersion equation for the transverse plasma shear flow is derived and applied to the determining of the saturation level of the instabilities. A review of sheared flows in ionosphere and laboratory experimental investigations is presented.

Keywords: <Enter Keywords here>
PACS: 03.40.G, 52.35.K

INTRODUCTION

The existence of inhomogeneous flows and currents along and across magnetic fields is a fundamental characteristic of space plasmas. One important element of the ionospheric flows appears to be velocity shear. Tsunoda et al.[1] reported that strong thermal ion upwelling, i.e. ion heating, occurs in regions of finite velocity shear. Localized static (or quasi–static) electric fields both perpendicular [2, 3] and parallel [4] to the magnetic field were found to be co-located with waves and coherent structures as well as with enhanced ion temperatures [5], [6], [7], [8]. Compared to the homogeneous plasma case, sheared flows introduce significant modifications to plasma stability. In this report we present the results of the analytical research of the stability of the inhomogeneous plasma shear flows transverse and along with respect to magnetic field, grounded on the Vlasov-Poisson system of equation.

STABILITY OF THE INHOMOGENEOUS PLASMAS WITH TRANSVERSE INHOMOGENEOUS ELECTRIC FIELD.

In Ref.[9] we obtain the following renormalized local dispersion equation for Maxwellian plasma immersed into homogeneous magnetic and transverse inhomogeneous electric fields:

$$1+\delta\varepsilon_i(\mathbf{k},\omega)+\delta\varepsilon_e(\mathbf{k},\omega)$$

$$\begin{aligned}
&= 1+\frac{1}{k^2\lambda_{Di}^2}\int_0^\infty dw_\perp w_\perp \int_{-\infty}^{\infty} dv_z \frac{1}{\sqrt{2\pi}v_{Ti}^3}\sum_{n=-\infty}^{\infty} J_n^2\left(\frac{k_\perp w_\perp}{\omega_{ci}}\right)\exp\left(-\frac{1}{2}\frac{v_z^2}{v_{Ti}^2}-\frac{1}{2}\frac{w_\perp^2}{v_{Ti}^2}\right)\\
&\times\left[-in\omega_{ci}\sqrt{\eta_i(X)}-\frac{ik_y v_{Ti}^2}{\omega_{ci}\eta_i(X)}\left(\frac{V_E''(X)}{2\omega_{ci}\eta_i(X)}-\frac{d\ln n_{0i}(X)}{dX}\right)-ik_z v_z\right]\\
&\int_0^\infty d\tau \exp\left[i\left(\omega-k_yV_E-k_zv_z-n\omega_{ci}\sqrt{\eta_i(X)}-\frac{k_yV_E''(X)w_\perp^2}{4\omega_{ci}^2\eta_i^2(X)}\right)\tau-C_0\tau-C_3\tau^3\right]\\
&+\frac{1}{k^2\lambda_{De}^2}\left(1+i\sqrt{\pi}z_{e*}W(z_{e0})A_{e0}\right)=0, \qquad (1)
\end{aligned}$$

where $\lambda_{D\alpha}=\sqrt{T_\alpha/4\pi n_\alpha e^2}$ is the Debye radius. The C_0 term describes the turbulent scattering of ions in plasma without shear flow, C_3 term describes the enhanced scattering of ions along the shear flow that resulted from the random transport of particles across the shear flow due to their interaction with turbulent electric field. The explicit expressions for C_0 and C_3 terms are obtained in Ref.[9]. On the initial linear stage of the instabilities development the terms C_0 and C_3 may be omitted and linear dispersion equation, which defines the linear stability properties of the inhomogeneous shear flow of the inhomogeneous plasma, is considered. We consider separately the case of the short wave length perturbations along the magnetic field, for which $k_z > k_y\left(V_E\rho_i^2\right)/\left(v_{Ti}L_v^2\eta_i^2(X)\right)$, and the opposite to that the case of the long wave length along the magnetic field. In the first case the linear dispersion equation has a form

$$1+\frac{1}{k^2\lambda_{Di}^2}\left\{1+\frac{i\sqrt{\pi}}{\sqrt{2}k_z v_{Ti}}\sum_{n=-\infty}^{\infty}W\left(\frac{\omega-k_yV_E(X)-n\omega_{ci}\sqrt{\eta_i(X)}}{\sqrt{2}k_z v_{Ti}}\right)A_{in}\right.$$
$$\left.\times\left[\omega-k_yV_E(X)+\frac{k_y v_{Ti}^2}{\omega_{ci}\eta_i(X)}\left(\frac{V_E''(X)}{2\omega_{ci}\eta_i(X)}-\frac{1}{L_n}\right)\right]\right\}+\frac{1}{k^2\lambda_{De}^2}\left(1+i\sqrt{\pi}z_{e*}W(z_{e0})A_{e0}\right)=0, \tag{2}$$

where $A_{\alpha n}=A_{\alpha n}\left(k_\perp^2\rho_\alpha^2\right)=I_n\left(k_\perp^2\rho_\alpha^2\right)e^{-k_\perp^2\rho_\alpha^2}, z_{e0}=\omega-k_yV_E(X)/\sqrt{2}k_zv_{Te}, z_{e*}=\omega-k_yV_E(X)-\omega_{de}\sqrt{2}k_zv_{Te}$ and where $\omega_{de(i)}=k_ycT_{e(i)}/eB_0L_n$ is the electron (ion) diamagnetic drift frequency. In the opposite case for which $k_z<k_y\left(V_E\rho_i^2\right)/\left(v_{Ti}L_v^2\eta_i^2(X)\right)$, the dispersion equation has a form

$$1+\frac{1}{k^2\lambda_{Di}^2}\left(1-\sum_{n=-\infty}^{\infty}\left(1+\zeta_{i0}-\frac{2\omega_{ci}\eta_i(X)}{L_nV_E''(X)}\right)\left(\frac{1}{\zeta_{in}}A_{in}\left(k_\perp^2\rho_i^2\right)\right.\right.$$
$$\left.\left.-\frac{i\pi}{2}\mathrm{sign}\left(k_yV_E''(X)\right)\left(1+\mathrm{sign}\,\zeta_{in}\right)e^{-\zeta_{in}}J_n^2\left(\sqrt{2\zeta_{in}}k_\perp\rho_i\right)\right)\right)+\frac{1}{k^2\lambda_{De}^2}\left(1+i\sqrt{\pi}z_{e*}W(z_{e0})A_{e0}\right)=0, \tag{3}$$

suitable for short wave length with $k_\perp\rho_i\gg 1$, and the following form is suitable for long wave length perturbations $k_\perp\rho_i\ll 1$:

$$1+\frac{1}{k^2\lambda_{Di}^2}\left[1+\left(1+\zeta_{i0}-\frac{2\omega_{ci}\eta_i(X)}{L_nV_E''(X)}\right)e^{-\zeta_{i0}}\left(i\frac{\pi}{2}\mathrm{sign}\left(k_yV_E''(X)\right)\left(1+\mathrm{sign}\left(\zeta_{i0}\right)\right)-Ei\left(\zeta_{i0}\right)\right)\right]$$
$$+\frac{1}{k^2\lambda_{De}^2}\left(1+i\sqrt{\pi}z_{e*}W(z_{e0})\right)=0, \tag{4}$$

where $\zeta_{in}=\left(\omega-k_yV_E-n\omega_{ci}\sqrt{\eta_i(X)}2\eta_i^2(X)\right)/\left(k_yV_E''(X)\rho_i^2\right)$ and $\zeta_{i0}=2\left(\omega-k_yV_E\right)\eta_i^2(X)/k_y\rho_i^2V_E''(X)$ and values $|\zeta_{in}|\gg 1$, which corresponds to small damping of waves on ions, are considered. The equations (2), (3), (4) are basic ones in our studies of the stability of the inhomogeneous transverse shear flow against the excitation of ion cyclotron and drift instabilities.

Ion cyclotron instabilities

We consider short wavelength waves, $k\rho_i\gg 1\gg k\rho_e$, which propagate almost across the magnetic field, such as $|z_{e0}|<1<|z_{in}|$. The frequency and the growth rate of the instability are determined by the relations

$$\widetilde{\omega}(\mathbf{k})\approx n\omega_{ci}\sqrt{\eta_i(X)}+\delta\omega(\mathbf{k})=n\omega_{ci}\sqrt{\eta_i(X)}+\frac{\left(n\omega_{ci}\sqrt{\eta_i(X)}+\Delta\right)A_{in}\left(k_\perp^2\rho_i^2\right)}{\left(1+\left(1/k^2\lambda_{Di}^2\right)+\left(T_i/T_e\right)\right)}, \tag{5}$$

$$\gamma\approx-\frac{\left(\widetilde{\omega}+\Delta\right)^2A_{in}\left(k_\perp^2\rho_i^2\right)}{\left(1+\left(1/k^2\lambda_{Di}^2\right)+\left(T_i/T_e\right)\right)^2}\sqrt{\frac{\pi}{2}}\frac{e^{-z_{in}^2}}{k_zv_{Ti}}-\sqrt{\frac{\pi}{2}}\frac{T_i}{T_e}\frac{\left(\widetilde{\omega}+\Delta\right)\left(n\omega_{ci}\sqrt{\eta_i(X)}-\omega_{de}\right)A_{in}\left(k_\perp^2\rho_i^2\right)}{\left(1+\left(1/k^2\lambda_{Di}^2\right)+\left(T_i/T_e\right)\right)^2k_zv_{Te}}. \tag{6}$$

where $\widetilde{\omega}=\omega-k_yV_E(X), \Delta=\left(k_yV_E''(X)\rho_i^2/2\eta_i^2\right)-\left(\omega_{di}/\eta_i(X)\right)$. It follows from Eq.(6) that the instability is in fact the drift-cyclotron instability modified by transverse shear flow. This instability is excited due to the inverse electron Landau damping of the short wavelength ion cyclotron waves with the frequency (5) and with $k_y\rho_i>$

$(T_i/T_e)\left(L_n\sqrt{\eta_i(X)}/\rho_i\right)\gg 1$ in the inhomogeneous plasma when $|n\omega_{ci}\sqrt{\eta_i(X)}|<|\omega_{de}|$. It follows from the equation (3) that the frequency $\omega(k)$ is still determined by the equation (5), but the growth rate is different and is equal to $\gamma=\gamma_i+\gamma_e$, where, approximately,

$$\gamma_i\approx-\pi\frac{(\delta\omega(k))^2}{|k_yV_E''|}\frac{\eta_i^2(1+\mathrm{sign}(\zeta_{in}))}{\rho_i^2A_{in}}e^{-\zeta_{in}}J_n^2\left(\sqrt{2\zeta_{in}}k_\perp\rho_i\right) \tag{7}$$

and

$$\gamma_e\approx-\frac{\delta\omega(k)\sqrt{\pi}A_{e0}}{1+(T_i/T_e)+k^2\lambda_{Di}^2}\frac{T_i}{T_e}\exp\left(-\left(\frac{n\omega_{ci}\sqrt{\eta_i}}{\sqrt{2}k_zv_{Te}}\right)^2\right)\frac{(n\omega_{ci}\sqrt{\eta_i}-\omega_{de})}{\sqrt{2}k_zv_{Te}}. \tag{8}$$

In Eqs.(7) and (8) we define $\zeta_{in}=2\eta_i^2\delta\omega(k)/k_yV_E''\rho_i^2,\quad \zeta_{i0}=2n\omega_{ci}\eta_i^{5/2}/k_yV_E''\rho_i^2,\quad z_{e0}=n\omega_{ci}\eta_i^{1/2}/\sqrt{2}k_zv_{Te}$.

The results obtained prove, that the perpendicular shear flow is shown to be stabilizing for the ion cyclotron waves.

Drift instabilities

It follows from the equation (2) that the frequency and the growth rate of the instability are

$$\widetilde{\omega}=\frac{1}{1+k_\perp^2\rho_s^2}\left(\frac{1}{2}k_yV_E''(X)\frac{\rho_s^2}{\eta_i^2(X)}-\omega_{de}\right)\left(1-k_\perp^2\rho_i^2\right)\approx\frac{T_e}{T_i}\frac{\Delta}{1+k_\perp^2\rho_s^2}, \tag{9}$$

$$\gamma=-\widetilde{\omega}^2\sqrt{\frac{\pi}{2}}\frac{T_e}{T_i}\frac{e^{-z_{i0}^2}}{|k_zv_{Ti}|\left(1+k_\perp^2\rho_s^2\right)}\left(1+k^2\lambda_{Di}^2+\frac{T_i}{T_e}\right)-\widetilde{\omega}^2\sqrt{\frac{\pi}{2}}\frac{T_i}{T_e}\frac{1}{|k_zv_{Te}|\Delta}\left(\widetilde{\omega}+k_y\left|\frac{cT_e}{eB_0}\right|\frac{1}{L_n}\right), \tag{10}$$

where Δ is determined above. As it follows from these equations in the homogeneous or inhomogeneous plasmas the interaction of low-frequency waves with ions leads to their damping. Also, it follows from the equation (4) that the frequency and the growth rate of the instability are

$$\widetilde{\omega}=\frac{1}{2}\frac{k_y\rho_i^2V_E''(X)}{\eta_i^2(X)\left((T_i/T_e)+k^2\lambda_{Di}^2\right)}-\frac{\omega_{di}}{\eta_i(X)\left((T_i/T_e)+k^2\lambda_{Di}^2\right)}, \tag{11}$$

$$\begin{aligned}\gamma=\gamma_i+\gamma_e=&-\frac{\widetilde{\omega}^2\eta_i^2(X)\pi}{|k_yV_E''(X)|\rho_i^2}(1+\mathrm{sign}(\zeta_{i0}))e^{-\zeta_{i0}}\left(1+\frac{1}{(T_i/T_e)+k^2\lambda_{Di}^2}\right)\\&-\frac{T_i}{T_e}\frac{\widetilde{\omega}}{\left((T_i/T_e)+k^2\lambda_{Di}^2\right)}\sqrt{\frac{\pi}{2}}\frac{(\widetilde{\omega}-\omega_{de})}{k_zv_{Te}},\end{aligned} \tag{12}$$

where $\zeta_{i0}=2\widetilde{\omega}^2\eta_i^2(X)/k_y\rho_i^2V_E''(X)$. It follows from Eq.(12) that in the case considered the interaction of drift waves with ions leads to their damping. The excitation of the velocity shear modified drift instability is possible due to the inverse electron Landau damping.

Nonlinear stage and saturation of the instabilities

As the example of the application of the developed renormalized stability theory we consider here the stabilization of the drift-cyclorton instability modified by transverse shear flow. It was obtained in Ref.[9], that the stabilization of the instabilities resulted from the enhanced decorrelation due to the flow shear occurs when $C_3>\gamma(\delta\omega)^2$, where γ is total nonlinear growth rate of the instability. In Ref.[9] we obtained relatively simple relation for C_3

$$C_3\approx\frac{e^2}{m_i^2}\left(\frac{V_E'(X)}{\omega_{ci}\eta_i(X)}\right)^2\frac{1}{(2\pi)^3}\sum_{n_1=-\infty}^{\infty}\int d\mathbf{k}_1|\phi(\mathbf{k}_1)|^2k_y^2k_{1y}^2J_{n_1}^2\left(\frac{k_{1\perp}w_\perp}{\omega_{ci}}\right)\frac{C_3}{(\delta\omega(\mathbf{k}_1))^4}, \tag{13}$$

which admits to receive the order of value estimate of the electrostatic turbulence energy density W at the saturate state, which is attained due to the effect of the enhanced decorrelation by the flow shear. It is interesting to note that the relation (13) does not depend on the value of k_z and is applicable to both cases, of "short" and "long" waves along the magnetic field. For the drift - cyclotron instabilities considered above the upper bound of that level is estimated as

$$\frac{W}{n_{0i}T_i} \leq \frac{1}{(k_\perp \rho_i)^4}\left(\frac{\delta\omega\eta_i}{V_E'}\right)^2. \tag{14}$$

The transverse wave number $k_\perp$ for the drift-cyclotron instabilities considered is limited by the condition $k_\perp\rho_i \sim k_y\rho_i > (T_i/T_e)\left(L_n\sqrt{\eta_i(X)}/\rho_i\right) \gg 1$ and level (14) is equal to

$$\frac{W}{n_{0i}T_i} \leq \left(\frac{T_e}{T_i}\frac{\rho_i}{L_n}\right)^4\left(\frac{\delta\omega}{V_E'}\right)^2 \tag{15}$$

which strongly reduces in the cases of weak plasma density inhomogeneity and strong flow shear. Note, that in the case of weak flow shear, for which the condition $\delta\omega > C_0$ holds, the saturation of drift-cyclotron instability will be achieved on higher level, which is determined by the estimate

$$\frac{W}{n_{0i}T_i} \leq \frac{1}{(k_\perp \rho_i)^4} \sim \left(\frac{T_e}{T_i}\frac{\rho_i}{L_n}\right)^4. \tag{16}$$

INSTABILITIES IN MAGNETIC FIELD–ALIGNED SHEAR FLOW OF INHOMOGENEOUS PLASMA.

We obtain, that ion cyclotron and drift-cyclotron instabilities are destabilized by a field-aligned shear flow. Inverse ion cyclotron damping of ion cyclotron waves and inverse electron Landau damping of ion cyclotron waves are potential sources for the excitation of the ion cyclotron instabilities due to shear of the field-aligned flow. The growth rates of new kinetic and hydrodynamic ion cyclotron instabilities, which are excite due to slow shear appeared to be proportional to the parameter $k_yV_0'/k_z\omega_{ci}$. These instabilities disappear in the shearless plasma.

Thus, the flow shears in magnetized plasmas are found to be important for controlling both the drift-wave and ion-cyclotron instabilities, where the parallel and perpendicular shears play different roles in these instabilities. The parallel flow shear can excite the drift-wave and ion-cyclotron instabilities, while the perpendicular flow shear suppress these instabilities regardless of the sign of the shear.

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