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Ion-temperature-gradient sensitivity of the hydrodynamic instability caused by shear in the magnetic-field-aligned plasma flow

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The cross-magnetic-field (i.e., perpendicular) profile of ion temperature and the perpendicular profile of the magnetic-field-aligned (parallel) plasma flow are sometimes inhomogeneous for space and laboratory plasma. Instability caused either by a gradient in the ion-temperature profile or by shear in the parallel flow has been discussed extensively in the literature. In this paper, (1) hydrodynamic plasma stability is investigated, (2) real and imaginary frequency are quantified over a range of the shear parameter, the normalized wavenumber, and the ratio of density-gradient and ion-temperature-gradient scale lengths, and (3) the role of inverse Landau damping is illustrated for the case of combined ion-temperature gradient and parallel-flow shear. We find that increasing the ion-temperature gradient reduces the instability threshold for the hydrodynamic parallel-flow shear instability, also known as the parallel Kelvin-Helmholtz instability or the D'Angelo instability. We also find that a kinetic instability arises from the coupled, reinforcing action of both free-energy sources. For the case of comparable electron and ion temperature, we illustrate analytically the transition of the D'Angelo instability to the kinetic instability as (a) the shear parameter, (b) the normalized wavenumber, and (c) the ratio of density-gradient and ion-temperature-gradient scale lengths are varied and we attribute the changes in stability to changes in the amount of inverse ion Landau damping. We show that near a normalized wavenumber $k_{\perp}\rho_i$ of order unity (i) the real and imaginary values of frequency become comparable and (ii) the imaginary frequency, i.e., the growth rate, peaks. © 2014 AIP Publishing LLC. [<http://dx.doi.org/10.1063/1.4890297>]

I. INTRODUCTION

Shear in magnetic-field-aligned (i.e., parallel) plasma flow can be found in space,^{1–3} fusion,^{4–8} and laboratory^{9–16} plasma. Ionospheric regions of inhomogeneous parallel plasma flow can map magnetically to magnetospheric regions of large-scale field-aligned currents, resulting in broadband electrostatic noise in the auroral zone of geospace.¹

Experimental observations from a number tokamaks and stellarators^{4–8} have found large nearly sonic parallel sheared flows inside the last closed flux surface. Near-sonic parallel sheared flows are systematically observed in the far scrape-off layer (SOL) of the X-point divertor tokamaks JT-60⁴ and Alcator C-Mod⁵ tokamaks, in the limiter tokamak Tore Supra.⁶ These flows are deemed unstable^{17–19} against the development of the well known hydrodynamic D'Angelo instability.²¹ Schwander *et al.*¹⁹ showed that plasma might be unstable to the parallel-flow shear instability around limiters, as inferred from the experimental findings of Fenzi *et al.*,²⁰ thereby explaining local enhancements of turbulence and showing that according to the local linear stability criterion, stability is sensitive to core parallel rotation. Their work speaks to the interest that would be given to future numerical modelling of

experimentally relevant plasma conditions to assess both the properties of the transport coefficients associated with the parallel-flow shear-driven instability and the presence of free-energy that would support turbulence that arises from that instability. In this paper, using analytical theory and numerical analysis, we quantify the frequency and growth rate of the parallel-flow shear-driven instability over a range of the shear parameter, the normalized wavenumber, and the ratio of density-gradient and ion-temperature-gradient scale lengths and demonstrate that the flow-shear threshold for instability reduces as the ion-temperature gradient increases. We also illustrate the role of inverse Landau damping in this case of combined ion-temperature gradient and parallel-flow shear.

If the parallel-ion-flow shear is accompanied by inhomogeneous ion temperature, a hydrodynamic instability can develop into a kinetic instability as will be shown. The investigation of plasma stability in the presence of these two free-energy sources was initiated by Migliuolo in his investigations of the plasma sheet boundary layer²² in the Earth's magnetosphere. He found that kinetic instability arises by virtue of the coupled action of both parallel-velocity shear $V'_0(x)$ and an ion temperature gradient that reinforce each other. Inverse ion Landau damping is responsible for the combined ion-temperature-gradient flow-shear-driven (ITG-FSD) kinetic instability, a conclusion quite different from D'Angelo's conclusion for the homogeneous-temperature, hydrodynamic

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FSD instability in the framework of the two-fluid equations. The calculations in Ref. 22 involved taking the large argument ($z_i = \omega/\sqrt{2}k_z v_{Ti} \gg 1$) limit of the ion plasma dispersion function and examining marginal stability, without quantifying the maximum growth rate. The linear analysis of the parallel-flow shear stability in the presence of inhomogeneous ion temperature with application to the edge tokamak plasma was performed in Ref. 23. It was predicted that the roles of magnetic shear, trapped electrons, and toroidal curvature are negligible for the ITG–SFD kinetic instability and the behavior of the instability growth rate was calculated only in the neighborhood of marginal instability.

In this paper, we present results from a numerical and analytical investigation of the sensitivity of the hydrodynamic D’Angelo mode to the ion temperature gradient and we interpret the transition of the hydrodynamic instability to the ITG–SFD kinetic instability. Extending beyond the result for marginal stability of Ref. 23, we arrive at the approximate analytical solution of the dispersion relation for the parameters associated with the maximum value of the growth rate.

The paper is organized as follows: The basic equations are presented in Sec. II. In Sec. III, we analyze the effect of the finite ion temperature and ion temperature gradient on the hydrodynamic D’Angelo mode. In Sec. IV, we consider the ITG–SFD kinetic instability. The Conclusions are presented in Sec. V.

II. BASIC EQUATIONS

We consider a kinetic Vlasov-Poisson model of inhomogeneous, magnetic-field-aligned, single-ion-species, collisionless plasma flow with velocity $\mathbf{V}_0(X_\alpha) \parallel B_0 \mathbf{e}_z$. The Vlasov equation for the perturbation $f_\alpha = F_\alpha - F_{0\alpha}$ of the distribution function F_α with equilibrium function $F_{0\alpha}$ in guiding center coordinates in slab geometry, $X_\alpha = x + \frac{v_\perp}{\omega_{c\alpha}} \sin \phi$, $Y_\alpha = y - \frac{v_\perp}{\omega_{c\alpha}} \cos \phi$, where x and y are coordinates of the particle position, $\omega_{c\alpha}$ is the cyclotron frequency, has a form

$$\begin{aligned} \frac{\partial f_\alpha}{\partial t} - \omega_{c\alpha} \frac{\partial f_\alpha}{\partial \phi} + v_z \frac{\partial f_\alpha}{\partial z} \\ = \frac{e}{m_\alpha \omega_{c\alpha}} \frac{\partial \Phi}{\partial Y} \frac{\partial F_{0\alpha}}{\partial X} - \frac{e}{m_\alpha} \frac{\omega_{c\alpha}}{v_\perp} \frac{\partial \Phi}{\partial \phi} \frac{\partial F_{0\alpha}}{\partial v_\perp} + \frac{e}{m_\alpha} \frac{\partial \Phi}{\partial z} \frac{\partial F_{0\alpha}}{\partial v_z}. \end{aligned} \quad (1)$$

The perturbed electrostatic potential Φ is determined by the Poisson equation

$$\Delta \Phi(\mathbf{r}, t) = -4\pi \sum_{\alpha=i,e} e_\alpha \int f_\alpha(\mathbf{v}, \mathbf{r}, t) d\mathbf{v}_\alpha. \quad (2)$$

In what follows, $F_{0\alpha}$ is considered as the shifted Maxwellian distribution function for electrons and ions ($\alpha = i, e$)

$$F_{0\alpha} = \frac{n_{0\alpha}(X_\alpha)}{(2\pi v_{T\alpha}^2)^{3/2}} \exp \left[-\frac{v_\perp^2}{2v_{T\alpha}^2} - \frac{(v_z - V_0(X_\alpha))^2}{2v_{T\alpha}^2} \right], \quad (3)$$

assuming the inhomogeneity direction of the density and temperature of the sheared-flow species is along coordinate X_α , $v_{T\alpha} = (T_\alpha(X_\alpha)/m_\alpha)^{1/2}$ is the thermal velocity. The flow velocity of ions \mathbf{V}_0 is assumed to be equal to that of the electrons which is consistent with the fluid approximation used

for the Kelvin-Helmholtz instability, but inconsistent with including the development of current-driven instabilities. We consider here the idealized inhomogeneous-flow case of homogeneous parallel-velocity shear, i.e., $V_0(X_\alpha) = V_{00} + V'_0 X_\alpha$, where V_{00} is the spatially homogeneous part of the flow velocity, and $V'_0 = \text{const}$. In order to simplify the problem, a velocity \mathbf{v} usually transforms from the laboratory to a convecting frame of reference, $\mathbf{v} = \hat{\mathbf{v}} + V_0(X_\alpha) \mathbf{e}_z$. After that transformation, the Vlasov equation takes the form

$$\begin{aligned} \frac{\partial f_\alpha}{\partial t} - \omega_{c\alpha} \frac{\partial f_\alpha}{\partial \phi} + (\hat{v}_z + V_0(X_\alpha)) \frac{\partial f_\alpha}{\partial z} \\ = \frac{e}{m\omega_{c\alpha}} \frac{\partial \Phi}{\partial Y_\alpha} \left(\frac{\partial F_{0\alpha}}{\partial X_\alpha} - V'_0 \frac{\partial F_{0\alpha}}{\partial \hat{v}_z} \right) \\ - \frac{e}{m_\alpha} \frac{\omega_{c\alpha}}{v_\perp} \frac{\partial \Phi}{\partial \phi} \frac{\partial F_{0\alpha}}{\partial v_\perp} + \frac{e}{m_\alpha} \frac{\partial \Phi}{\partial z} \frac{\partial F_{0\alpha}}{\partial v_z}. \end{aligned} \quad (4)$$

The general approach to solving Eq. (4) is a Fourier transform over time and space coordinates with employing the local approximation. In that approximation, a weakly inhomogeneous plasma is assumed, for which $\rho_i \ll [L_n, L_{Ti}, L_v]$. Here, $\rho_i = v_{Ti}/\omega_{ci}$ is the ion thermal Larmor radius, $L_n = [d \ln n_0(X)/dX]^{-1}$ is the density gradient scale length, $L_{Ti} = [d \ln T_i(X_i)/dX_i]^{-1}$ is the ion temperature scale length, and $L_v = [d \ln V_0(X_i)/X_i]^{-1}$ is the parallel flow velocity gradient scale length. Also, it is assumed that perturbations take the modal form of plane waves as $\sim e^{-i\omega t + i\mathbf{k}\mathbf{r}}$, and the spatial plasma inhomogeneity does not affect the structure of the perturbations—all modes being considered have wavelength much smaller than the spatial scale length, i.e., $k_x L_n \gg 1$, $k_x L_{Ti} \gg 1$ and $k_x L_v \gg 1$.

For the velocity-shear case, the general approach employing the local approximation requires additional arguments and is otherwise inadequate. Similar to the case of perpendicular-velocity shear,^{24,25} perturbations in the case of parallel-velocity shear experience continuously increasing mode distortion that accumulates with time, causing the otherwise time-independent frequency and wavenumber of the perturbations to exhibit a nonmodal time-dependence in the perpendicular component of the wavenumber. This shear-induced time dependence of the perpendicular wavenumber results in finite-time validity for the time-independent local dispersion equation. Beyond the time interval associated with modal-approach validity, the evolving electrostatic potential becomes principally different from the modal one and is determined by the integral equation.^{24,25}

To estimate the time-dependent effect of the shearing the perturbations by the parallel shear flow, a Fourier transformation of Eq. (4) is performed over the guiding center coordinates, assuming the smallness of the wavelength and of the ion Larmor radius, noted above

$$\begin{aligned} \frac{\partial f_\alpha}{\partial t} - \omega_{c\alpha} \frac{\partial f_\alpha}{\partial \phi} + ik_z \hat{v}_z f_\alpha(v_\perp, \phi, \mathbf{k}, t) - V'_0 k_z \frac{\partial f_\alpha}{\partial k_x} \\ = \frac{ie}{m_\alpha} \sum_{n=-\infty}^{\infty} J_n \left(\frac{k_\perp v_\perp}{\omega_{c\alpha}} \right) e^{-in(\phi-\theta)} \Phi(\mathbf{k}, t) \\ \times \left[\frac{k_y}{\omega_{c\alpha}} \left(\frac{\partial F_{0\alpha}}{\partial X_\alpha} - V'_0 \frac{\partial F_{0\alpha}}{\partial \hat{v}_z} \right) + \frac{n\omega_{c\alpha}}{v_\perp} \frac{\partial F_{0\alpha}}{\partial v_\perp} + k_z \frac{\partial F_{0\alpha}}{\partial v_z} \right], \end{aligned} \quad (5)$$

where the relation

$$V'_0 \int_{-\infty}^{\infty} dX_\alpha X_\alpha \int_{-\infty}^{\infty} dz \frac{\partial f_\alpha}{\partial z} e^{-ik_x X_\alpha - ik_z z} = -V'_0 k_x \frac{\partial f_\alpha}{\partial k_x} \quad (6)$$

was used, and the spatially homogeneous part of flow velocity is eliminated from the problem by a simple Galilean transformation for the coordinate z ($z \Rightarrow z + V_{00}t$).

The parallel-velocity-shear effect on the wave pattern distortion is characterized by the term $-V'_0 k_x (\partial f_\alpha / \partial k_x)$. The solution $k_x + V'_0 t k_z = K_x$ of the characteristic equation for Eq. (5)

$$dt = -d\hat{k}_x / V'_0 \hat{k}_z, \quad (7)$$

where K_x as the integral of Eq. (7) is time independent, reveals, that $f_\alpha = f_\alpha(\mathbf{v}, K_x, k_y, k_z, t) = f_\alpha(k_x + V'_0 t k_z, k_y, k_z, t)$, i.e., the wave number components k_x and k_z have to evolve in time in such a way that $k_x + V'_0 t k_z$ remains time independent. Note that for the instabilities considered in this paper $|k_x| \gg |k_z|$, the term $V'_0 t k_z$ may be neglected during a long time period until $V'_0 t < |k_x/k_z|$, until which time the solution for f_α is of the ordinary modal form with separate time and wave number dependences. For that time, the general dispersion equation is valid in the local approximation, and is given by

$$\begin{aligned} 1 + k^2 \lambda_{Di}^2 + i\sqrt{\pi} \left(z_{i0} - \chi_i \left(1 - \frac{\eta_i}{2} \right) \right) \sum_{n=-\infty}^{\infty} W(z_{in}) A_{in}(k_\perp^2 \rho_i^2) \\ - \frac{k_y V'_0}{k_z \omega_{ci}} \left[1 + i\sqrt{\pi} \sum_{n=-\infty}^{\infty} z_{in} W(z_{in}) A_{in}(k_\perp^2 \rho_i^2) \right] \\ - \eta_i \chi_i \sum_{n=-\infty}^{\infty} z_{in} \left(1 + i\sqrt{\pi} z_{in} W(z_{in}) \right) A_{in}(k_\perp^2 \rho_i^2) \\ - \eta_i \chi_i \sum_{n=-\infty}^{\infty} i\sqrt{\pi} W(z_{in}) e^{-k_\perp^2 \rho_i^2} k_\perp^2 \rho_i^2 [I_n(k_\perp^2 \rho_i^2) - I'_n(k_\perp^2 \rho_i^2)] \\ + \frac{T_i}{T_e} \left(1 + i\sqrt{\frac{\pi}{2}} \frac{(\omega - k_y v_{de})}{k_z v_{Te}} W(z_e) \right) = 0. \quad (8) \end{aligned}$$

In Eq. (8), λ_{Di} is the ion Debye length, $A_{in}(k_\perp^2 \rho_i^2) = I_n(k_\perp^2 \rho_i^2) e^{-k_\perp^2 \rho_i^2}$, I_n is the modified Bessel function of order n , $z_{in} = (\omega - n\omega_{ci}) / \sqrt{2} k_z v_{Ti}$, $z_e = \omega / \sqrt{2} k_z v_{Te}$, $v_{d\alpha} = (v_{T\alpha}^2 / \omega c_\alpha) (d \ln n_0(x) / dx)$ is the diamagnetic drift velocity of ions ($\alpha = i$) and electrons ($\alpha = e$), $\chi_i = k_y v_{di} / \sqrt{2} k_z v_{Ti}$, $\eta_i = d \ln T_i / d \ln n_i$ which is equal approximately to L_{T_i} / L_n , $W(z) = e^{-z^2} (1 + (2i/\sqrt{\pi}) \int_0^z e^{t^2} dt)$ is the complex error function. We consider low frequency modes with frequency ω much less than the ion cyclotron frequency ω_{ci} in the limit $|\omega| \lesssim k_z v_{Te}$ as is appropriate for velocity-shear and temperature-gradient instabilities. For these conditions, the general dispersion equation that accounts for parallel-flow shear and inhomogeneous profiles of both ion density and ion temperature and that accounts for the effects of thermal motion of ions, both along and across the magnetic field, is²²

$$\begin{aligned} 1 + \frac{T_i}{T_e} (1 + \Delta \varepsilon_e(\mathbf{k}, \omega)) - \left(\frac{k_y V'_0}{k_z \omega_{ci}} + z_i \eta_i \chi_i \right) A_{0i}(k_\perp^2 \rho_i^2) \\ + i\sqrt{\pi} W(z_i) \left\{ \left[z_i \left(1 - \frac{k_y V'_0}{k_z \omega_{ci}} \right) - \chi_i \left(1 - \frac{\eta_i}{2} (1 - 2z_i^2) \right) \right] \right. \\ \left. \times A_{0i}(k_\perp^2 \rho_i^2) + \chi_i \eta_i k_\perp^2 \rho_i^2 (A_{0i}(k_\perp^2 \rho_i^2) - A_{1i}(k_\perp^2 \rho_i^2)) \right\} = 0, \quad (9) \end{aligned}$$

where $z_i = z_{0i}$, $\Delta \varepsilon_e(\mathbf{k}, \omega) = i\sqrt{\pi} W(z_e)(z_e - \chi_e)$.

Usually,^{22,23} Eq. (9) is solved analytically in the asymptotic case of large argument z_i of the W function that corresponds to the exponentially small ion Landau damping. Justifiably, the smallness of the ion Landau damping is the necessary condition either for instability that arises from inverse electron Landau damping, for which $|z_e| \lesssim 1$, or for instability of the hydrodynamic type, for which $|z_e| \gg 1$. It was shown²² that for the combined case of parallel-velocity shear and inhomogeneous ion temperature, the kinetic instability arises from inverse ion Landau damping. The growth rate for that kinetic instability was determined^{22,23} for near-threshold, i.e., marginal-stability, conditions and for the case $z_i \gg 1$. Marginal-stability predictions do not necessarily correspond to the typically observed cases of growing waves or saturated-amplitude waves and to the concomitant anomalous transport phenomena, which are determined primarily by the perturbations which have maximum growth rate. Understanding that, in the present paper, we apply different approaches to the investigation of Eq. (9). Starting from the established theory of the hydrodynamic D'Angelo instability²¹ (which is known also as a parallel Kelvin-Helmholtz instability), and accounting for the effects of the finite ion Larmor radius, we analytically determine the predictive limits of the applying the basic assumption $|z_i| \gg 1$ in the theory of that instability. The violation of that assumption corresponds to the transition of the hydrodynamic instability to the instability of the kinetic type, for which inverse ion Landau damping becomes the dominant instability process. That analysis is accompanied by the comprehensive numerical investigation of Eq. (9) with particular intent to ascertain the role the thermal effects of ions having inhomogeneous temperature profile on plasma stability in the presence of parallel-velocity shear. The particular attention is paid to the parallel shearing flows having comparable ion and electron temperatures, which is the case relevant to tokamak edge-layer and to space plasma. An approximate expression for the maximum growth rate is obtained and the expression's validity is confirmed numerically.

III. HYDRODYNAMIC IONS

In the long-parallel-wavelength limit, $|z_i| \gg 1$, in which ion Landau damping is negligible, the dispersion equation (9) reduces to the form

$$\begin{aligned}
& 1 + \frac{T_i}{T_e} (1 + \Delta \varepsilon_e(\mathbf{k}, \omega)) - \left[1 + \left(1 - \frac{k_y V'_0}{k_z \omega_{ci}} \right) \frac{k_z^2 v_{Ti}^2}{\omega^2} \right] A_{0i} \\
& + \frac{k_y v_{di}}{\omega} \left(1 + (1 + \eta_i) \frac{k_z^2 v_{Ti}^2}{\omega^2} \right) A_{0i} \\
& - \eta_i \frac{k_y v_{di}}{\omega} \left(1 + \frac{k_z^2 v_{Ti}^2}{\omega^2} \right) k_{\perp}^2 \rho_i^2 (A_{0i} - A_{1i}) = 0, \quad (10)
\end{aligned}$$

where $A_{0,1i} \equiv A_{0,1i}(k_{\perp}^2 \rho_i^2)$.

For plasma without parallel-flow shear, this equation describes the hydrodynamic ion temperature gradient drift instability and the kinetic ion temperature gradient drift instability developed due to the inverse electron Landau damping of the drift waves when $k_z v_{Ti} \ll \omega \lesssim k_z v_{Te}$. In the presence of sheared plasma flow, this equation in the case $1 - k_y V'_0 / k_z \omega_{ci} > 0$ determines the shear-modified ion acoustic instability,^{26,27} which can be excited due to the inverse electron Landau damping for wide range of ion-electron temperature ratios even for ion-electron temperature ratios of the order of unity and larger. In this paper, we consider the case with $k_y V'_0 / k_z \omega_{ci} > 0$ in which the hydrodynamic D'Angelo instability²¹ develops with frequency $\omega(\mathbf{k})$

$$\omega(\mathbf{k}) = -k_y v_{di} \frac{(A_{0i} - \eta_i k_{\perp}^2 \rho_i^2 (A_{0i} - A_{1i}))}{2 \left(1 + \frac{T_i}{T_e} - A_{0i} \right)}, \quad (11)$$

and with the growth rate $\gamma(\mathbf{k})$,

$$\begin{aligned}
\gamma(\mathbf{k}) = & \frac{1}{\left(1 + \frac{T_i}{T_e} - A_{0i} \right)} \left[A_{0i} \left(\frac{k_y V'_0}{k_z \omega_{ci}} - 1 \right) k_z^2 v_{Ti}^2 \right. \\
& \times \left. \left(1 - A_{0i} + \frac{T_i}{T_e} \right) - \frac{k_y^2 v_{di}^2}{4} B_i^2(k_{\perp}^2 \rho_i^2, \eta_i) \right]^{1/2}, \quad (12)
\end{aligned}$$

where

$$B_i(k_{\perp}^2 \rho_i^2, \eta_i) = A_{0i} - \eta_i k_{\perp}^2 \rho_i^2 (A_{0i} - A_{1i}). \quad (13)$$

Equations (11) and (12) account for the thermal motion of ions across the magnetic field. We assume, in what follows, that $|k_y V'_0 / k_z \omega_{ci}| \ll m_i / m_e$. Under that condition, we have $|z_e| \ll 1$, and the term $\Delta \varepsilon_e(\mathbf{k}, \omega)$, which determines the effect of electron Landau damping, may be neglected in Eq. (12). The negative term containing the ion diamagnetic drift velocity in Eq. (12) indicates that plasma density inhomogeneity acts as a stabilizing factor for the hydrodynamic D'Angelo instability, whereas the $B_i(k_{\perp}^2 \rho_i^2, \eta_i)$ term, representing the effect of ion-temperature inhomogeneity, reduces the density gradient's stabilizing effect by reducing the magnitude of B_i^2 and reinforces the development of the hydrodynamic D'Angelo instability.

The D'Angelo instability with growth rate (12) occurs for the values of k_z bounded by the region $k_{z1} > k_z > k_{z2}$ in the (k_{\perp}, k_z) plane, where $k_{z1,2}$ is equal to

$$k_{z1,2} = \frac{k_y V'_0}{2 \omega_{ci}} \left(1 \pm \sqrt{1 - \frac{v_{di}^2}{(\rho_i V'_0)^2} G_i(k_{\perp}^2 \rho_i^2, \eta_i)} \right) \quad (14)$$

with

$$G_i(k_{\perp}^2 \rho_i^2, \eta_i) = \frac{B_i^2(k_{\perp}^2 \rho_i^2, \eta_i)}{A_{0i} \left(1 + \frac{T_i}{T_e} - A_{0i} \right)}. \quad (15)$$

It follows from Eq. (14) that such interval exists for velocity shearing rate above the threshold value, $V'_0 > \omega_{ci}(\rho_i / L_n) \sqrt{G_i}$.

The growth rate (12), as a function of $k_z \rho_i$, attains its maximum between k_{z1} and k_{z2} , at $k_z = (V'_0 / 2 \omega_{ci}) k_y$, and at values of k_y for which the function $B_i(k_{\perp}^2 \rho_i^2, \eta_i)$ vanishes for the given value of η_i . The maximum growth rate is equal to

$$\gamma(\mathbf{k}) = k_z v_{Ti} \sqrt{\frac{A_{0i}}{1 + \frac{T_i}{T_e} - A_{0i}}}. \quad (16)$$

The hydrodynamic treatment for the D'Angelo instability is valid when the condition $|z_i| > 1$ holds for the whole interval $k_{z1} > k_z > k_{z2}$. Because the growth rate (12) is greater than the frequency (11), when D'Angelo instability develops, $|z_i|$ may be expressed approximately as

$$|z_i| \approx \frac{\gamma}{\sqrt{2} k_{z1} v_{Ti}} = \sqrt{\frac{A_{0i}}{2 \left(1 + \frac{T_i}{T_e} - A_{0i} \right)}}. \quad (17)$$

It follows from Eq. (17) that for the comparable temperatures of the ions and electrons, which is the case relevant to space, Q-machine, and tokamak plasmas, and/or for the perturbations with wavelength of the order of the ion Larmor radius, $k_{\perp} \rho_i \sim 1$, we have $|z_i| \leq 1$ (and therefore $|z_e| \ll 1$) associated with the maximum growth rate, and the ion kinetic effects, such as ion Landau damping and finite-ion-Larmor-radius effects, significantly influence the growth rate. In Fig. 1 we present the plot of $|z_i|$ versus $k_{\perp} \rho_i$ for different values of the relation of ion to electron temperatures. It displays, that only for the ion temperature less than the electron temperature the hydrodynamic approximation $|z_i| \gg 1$ is valid.

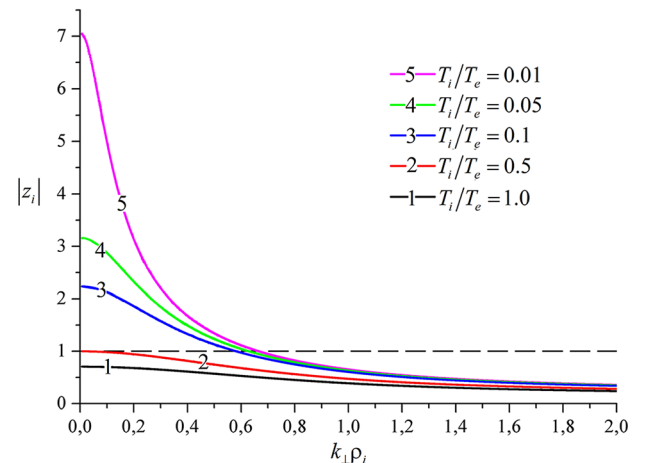


FIG. 1. The argument $|z_i|$ of the plasma dispersion function for the maximum growth rate (16) of the D'Angelo hydrodynamic instability versus $k_{\perp} \rho_i$. The dashed line corresponds to $|z_i| = 1$. Below that line, Eq. (12) for the hydrodynamic growth rate is not valid.

Numerical solution to Eq. (9) is necessary because simplifying the dispersion equation (9) eliminates the ability to properly account for the ion kinetic effects. The results of the numerical solution of Eq. (9), which confirm the importance of the ion kinetic effects, are presented on Figs. 2–5. Parameters considered pertinent to the conditions of SOL of Tokamaks: $\rho_i/L_n = 0.002$, $v_{di}/v_{Ti} = -0.002$. Also, we assume that $k_x = k_y = k_\perp/\sqrt{2}$. In Fig. 2, we present the plots of normalized frequency ω/ω_{ci} , normalized growth rate γ/ω_{ci} , $|z_i|$, and $|z_e|$ as a function $k_\perp\rho_i$ for $V'_0/\omega_{ci} = 0.001$, $T_i/T_e = 1.0$, and $(k_z\rho_i)^{-1} = 1800$ for different values of the parameter η_i . The main result of Fig. 2 is that the maximum growth rate is obtained in the region $k_\perp\rho_i \sim 1$, where $|z_i| \lesssim 1$. It is worth

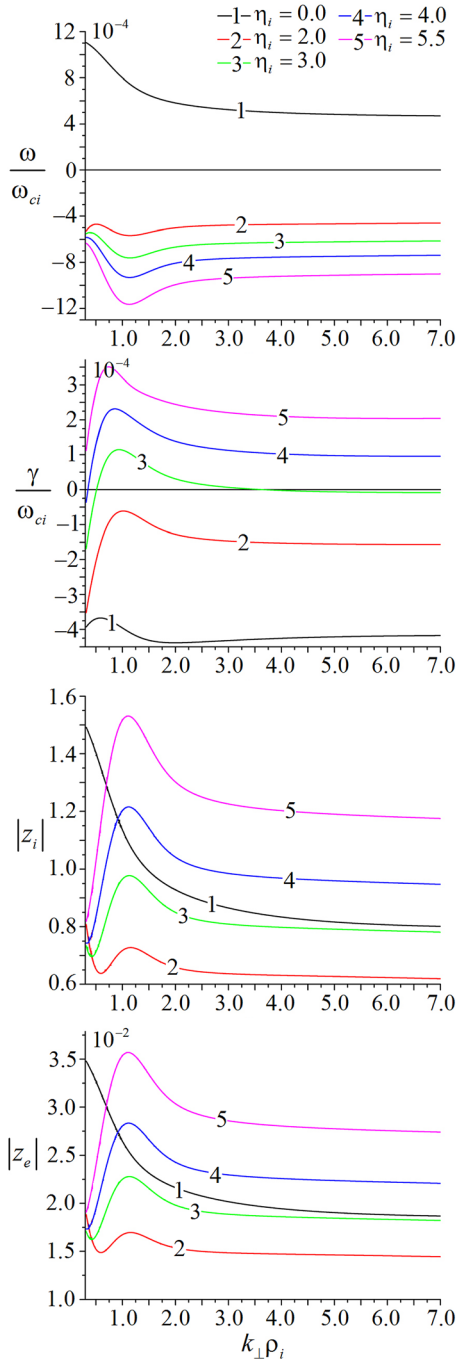


FIG. 2. The normalized frequency ω/ω_{ci} , normalized growth rate γ/ω_{ci} , $|z_i|$, and $|z_e|$ versus $k_\perp\rho_i$ for $V'_0/\omega_{ci} = 0.001$, $T_i/T_e = 1.0$, and $(k_z\rho_i)^{-1} = 1800$.

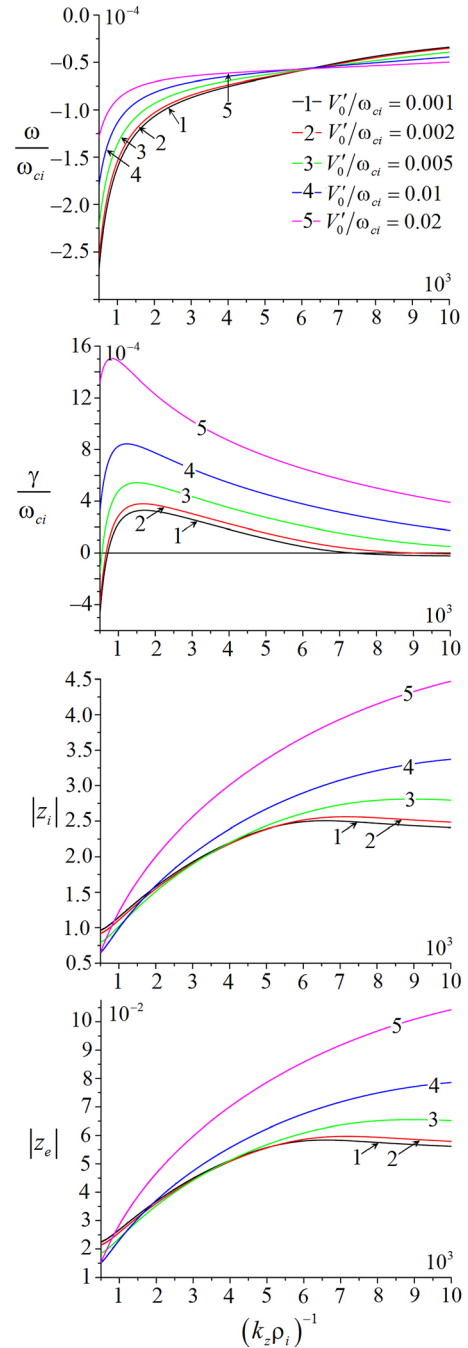


FIG. 3. The normalized frequency ω/ω_{ci} , normalized growth rate γ/ω_{ci} , $|z_i|$, and $|z_e|$ versus $k_z\rho_i$ for $\eta_i = 3.0$, $T_i/T_e = 1.0$, and $k_\perp\rho_i = 0.9$.

noting that for the case of negligible parallel-flow shear shown in Fig. 2, the effect of the ion temperature gradient, η_i , is pronounced.

The plots of normalized frequency ω/ω_{ci} , normalized growth rate γ/ω_{ci} , $|z_i|$, and $|z_e|$ as a function of $(k_z\rho_i)^{-1}$ for parameters $\eta_i = 3.0$, $T_i/T_e = 1.0$, and $k_\perp\rho_i = 0.9$, for which the instability was predicted by Fig. 2, are presented in Figs. 3 and 4. We find that the region of the maximum growth rate corresponds to $(k_z\rho_i)^{-1} \sim 2000$. It is interesting to note that we obtain $|z_i| \sim 1$ for that region.

Fig. 5 shows the normalized frequency ω/ω_{ci} , normalized growth rate γ/ω_{ci} , $|z_i|$, and $|z_e|$ as a function of $(V'_0/\omega_{ci})^{-1}$ for different values of η_i . The parameters

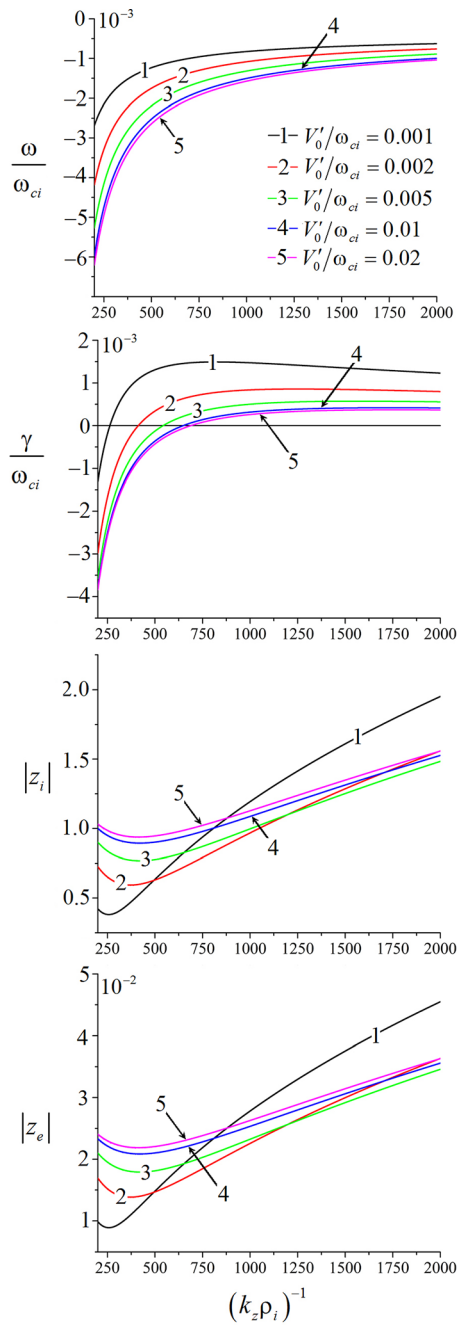


FIG. 4. The normalized frequency ω/ω_{ci} , normalized growth rate γ/ω_{ci} , $|z_i|$, and $|z_e|$ versus $(k_z \rho_i)^{-1}$ between 200 and 2000, for $\eta_i = 3.0$, $T_i/T_e = 1.0$, and $k_\perp \rho_i = 0.9$.

$k_\perp \rho_i \sim 1$, $(k_z \rho_i)^{-1} \sim 2000$ were used, at which the growth rate attains maximal value according to Figs. 2–4. We find that the instability requires the presence of the ion temperature gradient η_i when the value of velocity shear is small, consistent with the interpretation of Fig. 2. Common to Figs. 2–5, $|z_i| \sim 1$ in the region of maximum growth rate.

It follows from Fig. 5 that at small values of $(V'_0/\omega_{ci})^{-1}$, we have $|z_i| > 1$, independent of parameter η_i , and the D’Angelo instability²⁸ develops. As $(V'_0/\omega_{ci})^{-1}$ increases, and at small values of η_i parameter, the growth rate decreases and eventually becomes negative. At $\eta_i \geq 3$, the growth rate is positive and no longer depends on the sign or magnitude of $(V'_0/\omega_{ci})^{-1}$.

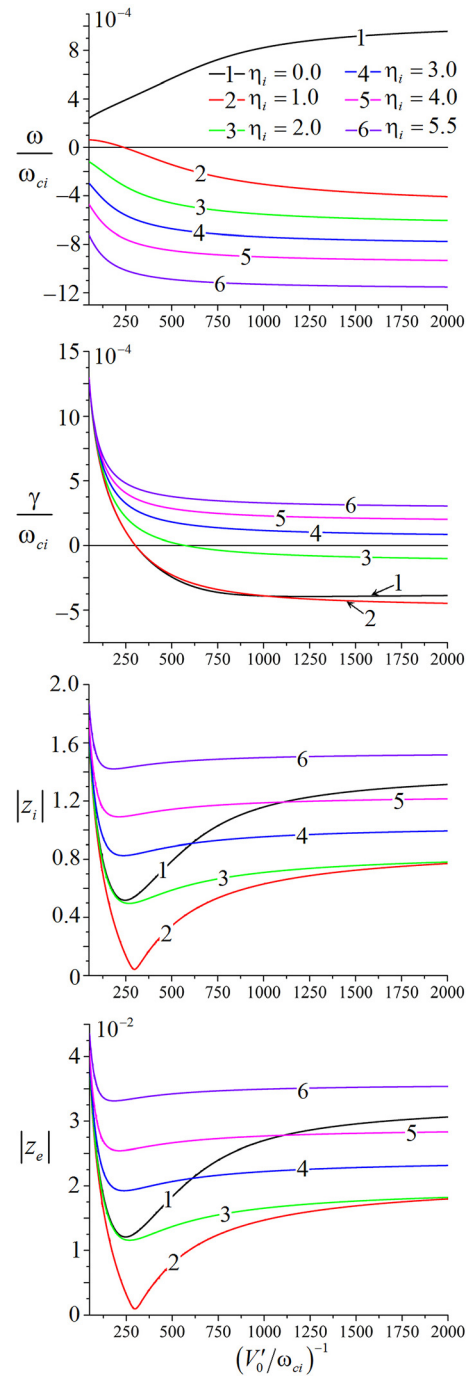


FIG. 5. The normalized frequency ω/ω_{ci} , normalized growth rate γ/ω_{ci} , $|z_i|$, and $|z_e|$ versus $(V'_0/\omega_{ci})^{-1}$ for $k_\perp \rho_i = 0.9$, $T_i/T_e = 1.0$, and $(k_z \rho_i)^{-1} = 1800$.

IV. THE KINETIC, COMBINED ION-TEMPERATURE-GRADIENT PARALLEL-FLOW SHEAR-DRIVEN (ITG–SFD) INSTABILITY

Extending beyond the near-marginal-stability analysis of Rogister *et al.*,²³ we consider the dispersion properties of the ITG–SFD instability for $|z_i|$ comparable to unity, values at which the growth rate of ITG–SFD instability is maximum. For these values of $|z_i|$, we take advantage of a Pade approximation for $W(z_i)$ in the form

$$W(z_i) \approx \frac{\sqrt{\pi}}{\sqrt{\pi} - 2iz_i}, \quad (18)$$

and apply it to Eq. (9). As a result, we obtain the simplest approximate dispersion equation for the kinetic ITG–SFD instability for the $|z_i| \leq 1$ domain

$$\begin{aligned} z_i^2 A_{0i} \eta_i |\chi_i| (\pi - 2) + z_i \left[(\pi - 2) A_{0i} \left(1 - \frac{k_y V'_0}{k_z \omega_{ci}} \right) \right. \\ \left. - 2 \left(1 + \frac{T_i}{T_e} - A_{0i} \right) - i \sqrt{\pi} \eta_i |\chi_i| A_{0i} \right] \\ + \pi |\chi_i| \left[A_{0i} \left(1 - \frac{\eta_i}{2} \right) - \eta_i k_\perp^2 \rho_i^2 (A_{0i} - A_{1i}) \right] \\ - i \sqrt{\pi} \left(1 + \frac{T_i}{T_e} - A_{0i} \frac{k_y V'_0}{k_z \omega_{ci}} \right) = 0. \end{aligned} \quad (19)$$

This algebraic equation for z_i , in which all terms are assumed to be of the same order of value, is not more complicated analytically than the dispersion equation for the drift instabilities obtained in the opposite limit $|z_i| \gg 1$. It can be easily solved with solution

$$\begin{aligned} \frac{\omega_{1,2}}{\sqrt{2} k_z v_{Ti}} = - \frac{A_{0i} \pi \left(1 - \frac{k_y V'_0}{k_z \omega_{ci}} \right) - 2 \left(1 + \frac{T_i}{T_e} - A_{0i} \frac{k_y V'_0}{k_z \omega_{ci}} \right)}{2 A_{0i} \eta_i |\chi_i| (\pi - 2)} \\ \pm \sqrt{\frac{r+x}{2}}, \end{aligned} \quad (20)$$

$$\frac{\gamma_{1,2}}{\sqrt{2} k_z v_{Ti}} = \frac{\sqrt{\pi}}{2(\pi - 2)} \pm \sqrt{\frac{r-x}{2}}, \quad (21)$$

where $r = \sqrt{x^2 + y^2}$ and

$$\begin{aligned} x = \left[\frac{A_{0i} \pi \left(1 - \frac{k_y V'_0}{k_z \omega_{ci}} \right) - 2 \left(1 + \frac{T_i}{T_e} - A_{0i} \frac{k_y V'_0}{k_z \omega_{ci}} \right)}{2 A_{0i} \eta_i |\chi_i| (\pi - 2)} \right]^2 \\ - \frac{\pi}{4(\pi - 2)^2} - \frac{\pi}{(\pi - 2)} \left[\frac{2 - \eta_i}{2 \eta_i} - k_\perp^2 \rho_i^2 \left(1 - \frac{A_{1i}}{A_{0i}} \right) \right], \end{aligned} \quad (22)$$

$$\begin{aligned} y = - \frac{\sqrt{\pi}}{2(\pi - 2)} \left[\frac{1}{2 A_{0i} \eta_i |\chi_i| (\pi - 2)} \right. \\ \left. \times \left(A_{0i} \pi \left(1 - \frac{k_y V'_0}{k_z \omega_{ci}} \right) - 2 \left(1 + \frac{T_i}{T_e} - A_{0i} \frac{k_y V'_0}{k_z \omega_{ci}} \right) \right) \right. \\ \left. + 2 \frac{1 + \frac{T_i}{T_e} - A_{0i} \frac{k_y V'_0}{k_z \omega_{ci}}}{A_{0i} \eta_i |\chi_i|} \right]. \end{aligned} \quad (23)$$

The “quick” solution (20) and (21) with sign “+” yields for the growth rate satisfactory accuracy (within 5% relative error) compared with the numerical solution of Eq. (9) presented in Fig. 2, for $\eta_i \geq 2$ and any values of $V'_0/\omega_{ci} > 0$ in the region where the growth rate attains maximum value.

In the case of a plasma with homogeneous ion temperature, but with inhomogeneous density, i.e., for $\eta_i = 0$, this instability continues to exist in the short wavelength range as the kinetic D’Angelo instability,²⁸ which is excited due to

inverse ion Landau damping. In Ref. 28, the wave real frequency and the growth rate were obtained in the vicinity of the instability threshold. Equation (17) gives for $\eta_i = 0$ a simple expression for wave frequency and growth rate of the ion kinetic D’Angelo instability for the range over which the growth rate is maximum

$$\omega = \frac{\pi A_{0i} |k_y v_{di}|}{A_{0i} \left[(\pi - 2) \left(\frac{k_y V'_0}{k_z \omega_{ci}} - 1 \right) - 2 \right] + 2 \left(1 + \frac{T_i}{T_e} \right)}, \quad (24)$$

$$\gamma = \frac{\sqrt{2\pi} k_z v_{Ti} \left(A_{0i} \frac{k_y V'_0}{k_z \omega_{ci}} - \left(1 + \frac{T_i}{T_e} \right) \right)}{A_{0i} \left[(\pi - 2) \left(\frac{k_y V'_0}{k_z \omega_{ci}} - 1 \right) - 2 \right] + 2 \left(1 + \frac{T_i}{T_e} \right)}. \quad (25)$$

It follows from Eq. (25) that the instability exists for the $k_z \rho_i$ values below a certain value

$$k_z \rho_i < \frac{V'_0}{\omega_{ci}} \frac{A_{0i}}{1 + \frac{T_i}{T_e}} k_y \rho_i. \quad (26)$$

According to Fig. 6, the solution (21) is valid to an accuracy of better than 10% of the relative error $\varepsilon_\gamma = (\gamma_{approx} - \gamma_{exact})/\gamma_{exact}$ over the wide intervals of the pertinent parameters. In the case of the parallel-flow shear with homogeneous ion temperature²⁸ ($\eta_i = 0$), the relative error between the approximate solution (23) and exact numerical solution to Eq. (7) (black line on Fig. 6, case (c)) is less than 10% only for large shear, i.e., $(V'_0/\omega_{ci})^{-1} \leq 500$. The accuracy of the approximate solution (23) improves with increasing value of $\eta_i \neq 0$.

In Eqs. (20) and (21), it was assumed that $\eta_i \neq 0$. In the different case of zero η_i , zero V'_0 , and $T_i \sim T_e$, Eqs. (20) and (21) predict the absence of the kinetic instability with $|z_i| \leq 1$

$$\omega = \frac{\pi A_{0i} |k_y v_{di}|}{2 \left(1 + \frac{T_i}{T_e} \right) - \pi A_{0i}}, \quad (27)$$

$$\frac{\gamma}{\sqrt{2} k_z v_{Ti}} = - \frac{\sqrt{\pi} \left(1 + \frac{T_i}{T_e} \right)}{2 \left(1 + \frac{T_i}{T_e} \right) - \pi A_{0i}}. \quad (28)$$

V. DISCUSSIONS AND CONCLUSIONS

In this paper, we elucidated the thermal effects of ions having inhomogeneous temperature profile on plasma stability in the presence of parallel flow shear. On the basis of the numerical solution of the general dispersion equation (7), we confirmed that the kinetic instability develops jointly with hydrodynamic D’Angelo instability due to inverse ion Landau damping and has comparable real and imaginary values of frequency at short wavelength over the interval having $k_\perp \rho_i$ of order unity.

We find that increasing the ion-temperature gradient reduces the instability threshold for the hydrodynamic

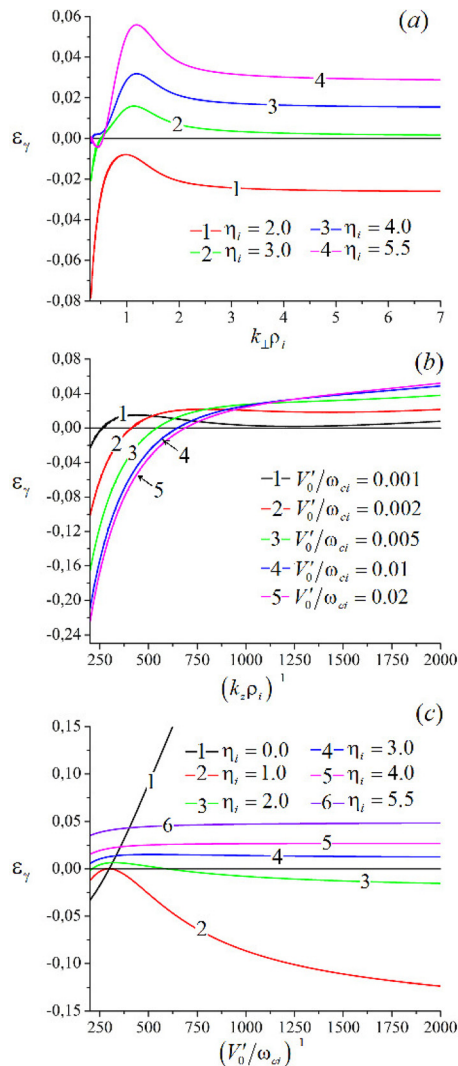


FIG. 6. The relative error ε_γ for the growth rate γ : (a) versus $k_\perp\rho_i$ for $V'_0/\omega_{ci} = 0.001$, $T_i/T_e = 1.0$, and $(k_\perp\rho_i)^{-1} = 1800$; (b) versus $(k_\perp\rho_i)^{-1}$ between 200 and 2000, for $\eta_i = 5.5$, $T_i/T_e = 1.0$, and $k_\perp\rho_i = 0.9$; (c) versus $(V'_0/\omega_{ci})^{-1}$ for $k_\perp\rho_i = 0.9$, $T_i/T_e = 1.0$, and $(k_\perp\rho_i)^{-1} = 1800$.

D'Angelo instability and that a kinetic instability arises from the coupled, reinforcing action of parallel-flow shear and ion-temperature gradient. For the case of comparable electron and ion temperature, we illustrate analytically the transition of the D'Angelo instability to the kinetic instability as the shear parameter, the normalized wavenumber, and the ratio of density-gradient and ion-temperature-gradient scale lengths are varied.

The approximate analytical solution of the dispersion equation, which uses a simple Pade approximation (16) for the complex error function, is reported for the parameters associated with the maximum growth rate. The approximate Pade solution of the dispersion equation was compared with the numerical solution of the dispersion equation for the same plasma conditions and for numerical parameters that may be pertinent to the conditions of the scrape-off-layer in tokamaks. We find that the thermal motion of ions along the magnetic field may be important for those plasma conditions and that improvement could be derived by incorporating the parallel dynamics of ions along the magnetic field into the

SOL codes. Although the dispersion equation (7) that accounted for the ion kinetic effects is the simplest one, it does not account for numerous effects, such as the 3D inhomogeneity of the toroidal magnetic field, presence of the limiters or divertors that lead to finite field-line length issues, or numerous other effects associated with the processes taking place in a tokamak or stellarator. Because of unexplored dependencies of the shear parameter on the realistic factors present in toroidal geometry, the presented plots provide only qualitative estimates for the frequency and growth rate for the D'Angelo instability in the SOL-edge layer having inhomogeneous ion temperature. For the less complicated geometry of space plasma, the estimates may prove to be more accurate. In both cases, intuition for interpreting instability and the behavior of unstable modes can be derived from the results presented here.

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