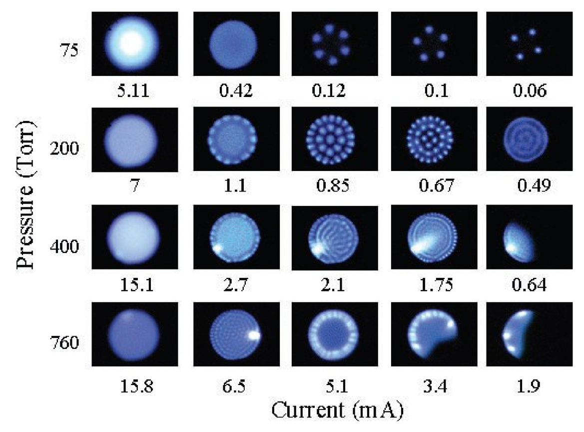
Stability of current transfer to cathodes of DC glow discharges

UNIVERSIDADE da MADEIRA
Departamento de Física

P. G. C. Almeida, M. S. Benilov, M. J. Faria, and V. V. Mikhailenko
Departamento de Física, Universidade da Madeira, Portugal

Motivation

- It is well known that current transfer to glow cathodes occurs in the abnormal discharge and in the normal discharge.
- Recently, observations of patterns of more than one spot in DC glow microdischarges have been reported.

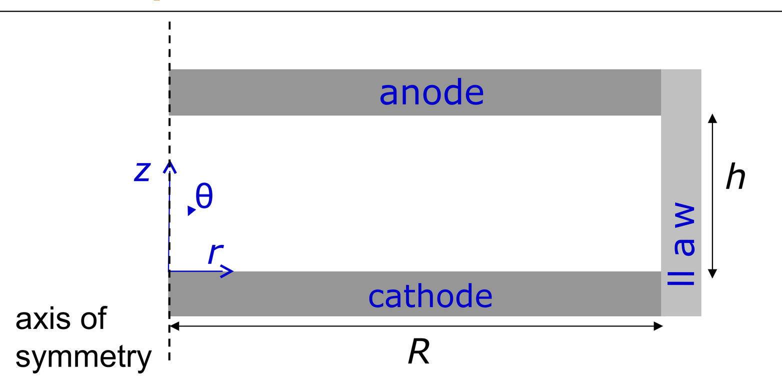


Self-organization in microdischarges in xenon. From K. H. Schoenbach, M. Moselhy and W. Shi 2004.

- It was hypothesized quite some time ago that **different modes are described by multiple solutions that should exist in the conventional theory of DC glow discharges**.
- In the framework of this approach, different modes of current transfer, including the normal discharge, stem from self-organization, which is an inherent property of current transfer to cathodes of DC discharges. **No special mechanism are required for self-organization!**
- The multiple solutions describing different modes were actually computed only very recently [1].
- In this contribution, **stability of different steady-state modes is analyzed numerically and analytically.**

Model

Geometry



Equations

$$\nabla\cdot\mathbf{J}_e = n_e\mu_e\alpha E - \beta n_e n_i,$$
$$\frac{\partial n_i}{\partial t} + \nabla\cdot\mathbf{J}_i = n_e\mu_e\alpha E - \beta n_e n_i,$$
$$\mathbf{J}_{i,e} = -D_{i,e}\nabla n_{i,e} \mp n_{i,e}\mu_{i,e}\nabla\varphi,$$
$$\varepsilon_0\nabla^2\varphi = -e\left(n_i - n_e\right).$$

i, e: ions, electrons

- The non-stationary term both in the electron conservation equation and the Poisson equation affect the calculated spectrum very weakly. This is an indication that the perturbations develop on a time scale governed by drift of ions.

Boundary conditions

Anode: $n_i = 0;\ \frac{\partial n_e}{\partial z} = 0;\ \varphi = U.$

Cathode: $\frac{\partial n_i}{\partial z} = 0;\ \mathbf{J}_e = -\gamma\,\mathbf{J}_i;\ \varphi = 0.$

Wall: $n_{i,e} = 0;\ j_r + \varepsilon_0\frac{\partial E_{R-0}}{\partial t} = 0.$

U: discharge voltage
j_r: density of radial conduction current in the plasma
E_{R-0}: electric field on the plasma side

- Physical meaning of the second boundary condition in the wall: zero of the density of total electric current on the plasma side, which implies a constant electric field in the dielectric wall.

Power supply

- The discharge is current-controlled.

Formalism of the linear stability theory

- A solution is sought in the form of a superposition of a steady-state solution and a small perturbation:

$$n_{i,e}(\mathbf{r},t) = n_{i0,e0}(r,z) + e^{\lambda t} n_{i1,e1}(\mathbf{r}) + \ldots,$$
$$\varphi(\mathbf{r},t) = \varphi_0(r,z) + e^{\lambda t}\varphi_1(\mathbf{r}) + \ldots,$$
$$U(t) = U_0 + e^{\lambda t} U_1 + \ldots$$
$$I(t) = I_0.$$

- Re λ < 0 for all perturbations: a stable state.
- Re λ > 0 for at least one perturbation: an unstable state.

Stability of axially symmetric steady states

- Perturbations of axially symmetric steady states are harmonic with respect to the azimuthal angle:

$$n_{i1,e1}(\mathbf{r}) = n_{i,e}^{(m)}(r,z)\begin{Bmatrix}\cos(m\theta)\\ \sin(m\theta)\end{Bmatrix},$$
$$\varphi_1(\mathbf{r}) = \varphi^{(m)}(r,z)\begin{Bmatrix}\cos(m\theta)\\ \sin(m\theta)\end{Bmatrix}.$$

m=0: axially symmetric perturbations
$m \neq 0$: 3D perturbations with period $2\pi/m$

- The perturbation problem governing stability against each of the harmonics may be formulated in 2D.
- The whole spectrum of stability is found by solving a series of 2D problems.

Numerical procedure

- The steady-state and perturbation problems are solved numerically with the use of the commercial software COMSOL Multiphysics.
- It has been found that the choice of the mesh affects the spectrum of stability stronger than it affects the steady-state results. The usual criterion was used:

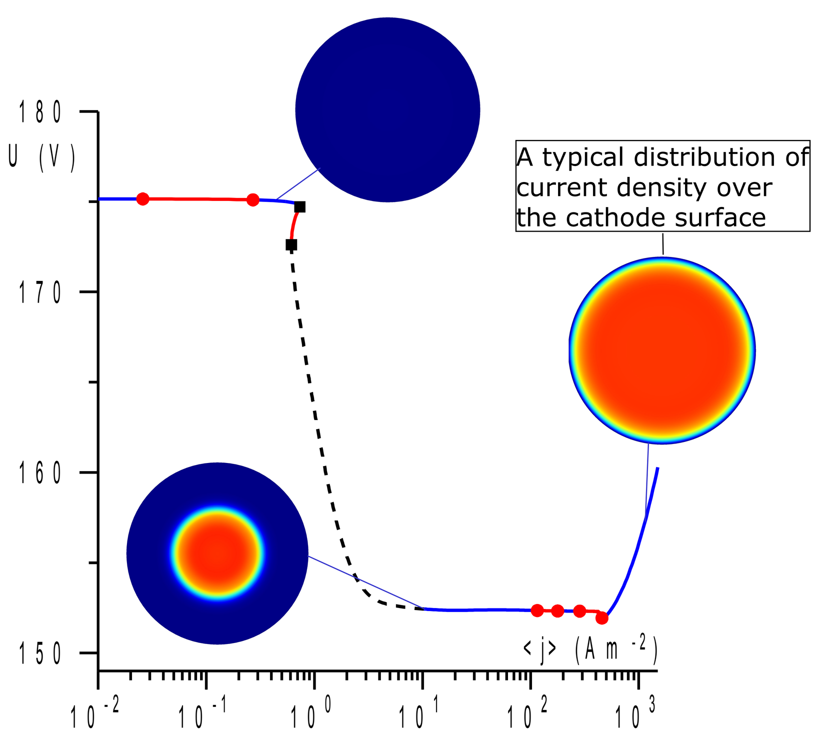
Variation of results from one mesh to another should be small, otherwise the results are deemed unreliable.

- **It turned out possible to obtain reliable results on steady-state characteristics of all steady-state modes of current transfer at all currents.**
- **It turned out possible to obtain reliable results on stability of steady-state modes in most cases.**

Steady-state solutions

Gas: Xe, p = 30 Torr;
R = 1.5 mm; h = 0.5 mm.

Fundamental mode

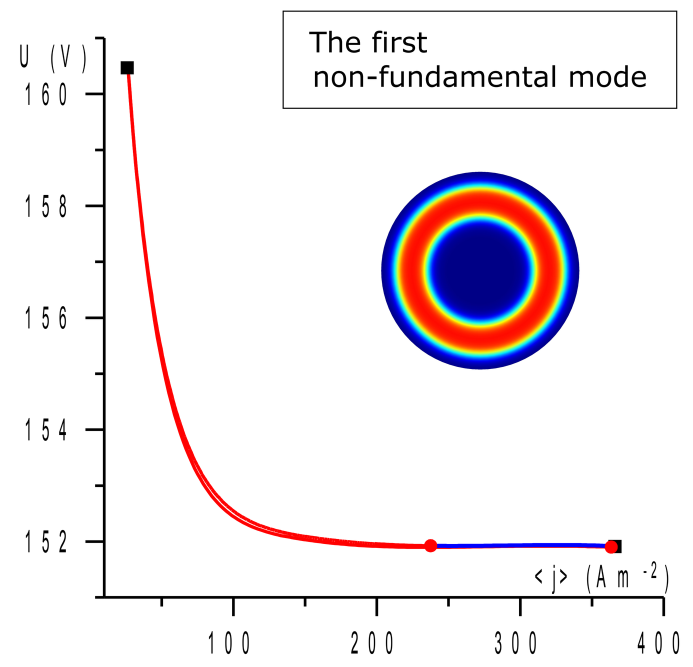


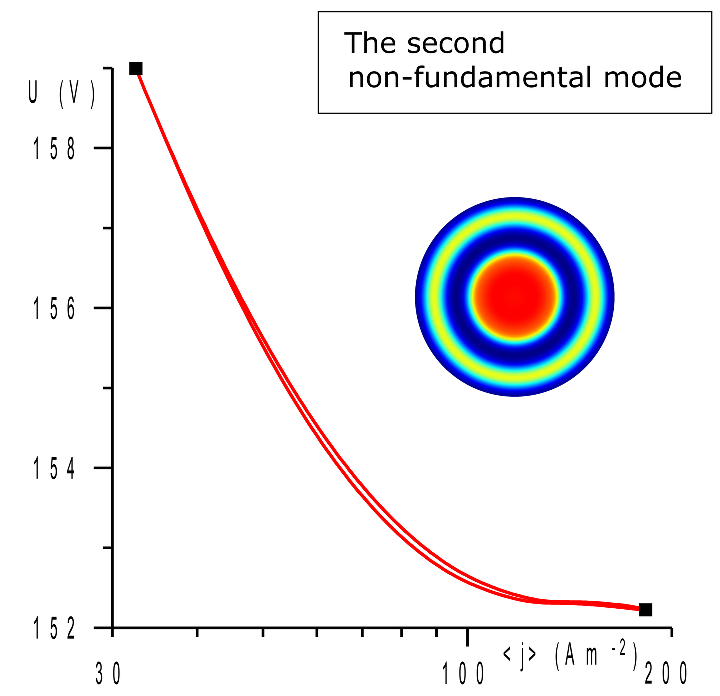
■ : turning points • : points of change of stability
—— : stable section —— : unstable section
-----: stability unknown

- The fundamental mode is the mode of highest symmetry admitted by the discharge which exists at all discharge currents: a 2D mode.
- This mode describes the abnormal discharge, the normal discharge, the subnormal discharge, and the Townsend discharge.

Non-fundamental modes

- Non-fundamental modes are those which exist in limited current ranges.
- Each mode represents a closed curve constituted by two branches separated by two turning points.
- Two non-fundamental modes exist in the conditions considered.





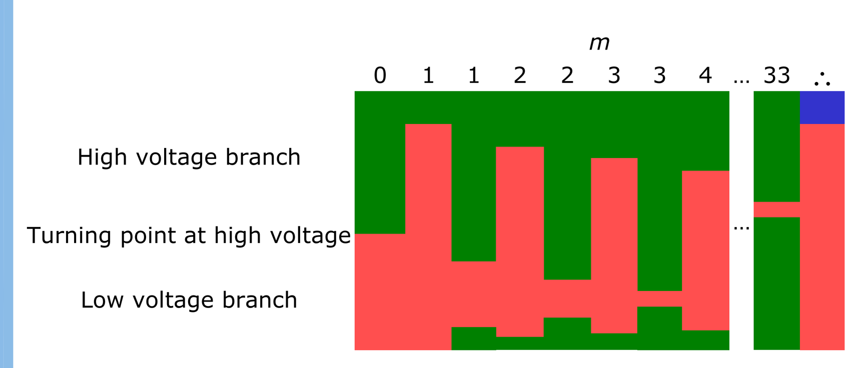
Stability of steady-state modes

- The **spectrum** of each type of perturbations of each steady-state mode comprises both **real and complex eigenvalues.** (For comparison: spectra are always real for the arc discharge [2].)

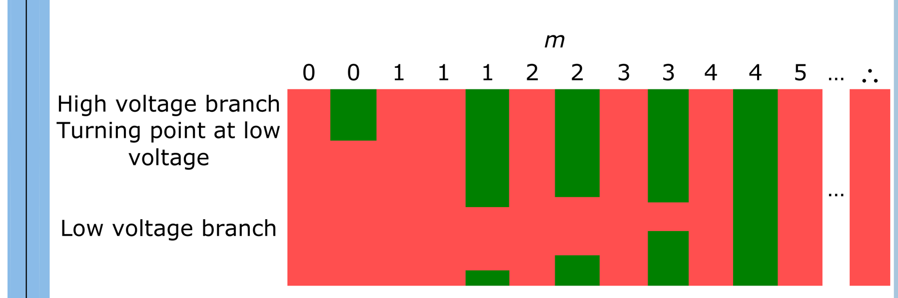
Fundamental mode

- The abnormal discharge is stable.
- A loss of stability occurs in the vicinity of the minimum of the CVC due to a **steady-state 3D perturbation with period 2π**. This conforms to the observations of a non-symmetric normal spot.
- There are two windows of stability in the normal discharge => **the possibility of observation of axially symmetric normal spots.**
- There is a window of instability in the Townsend discharge. The change of stability occurs due to an **axially symmetric oscillatory perturbation**. This conforms to observations (e.g., [4]) and modeling (e.g., [5]) that show oscillatory behaviour of the Townsend discharge.

Non-fundamental modes



Stability of the first non-fundamental mode against perturbations with period $2\pi/m$.



Stability of the second non-fundamental mode against perturbations with period $2\pi/m$.

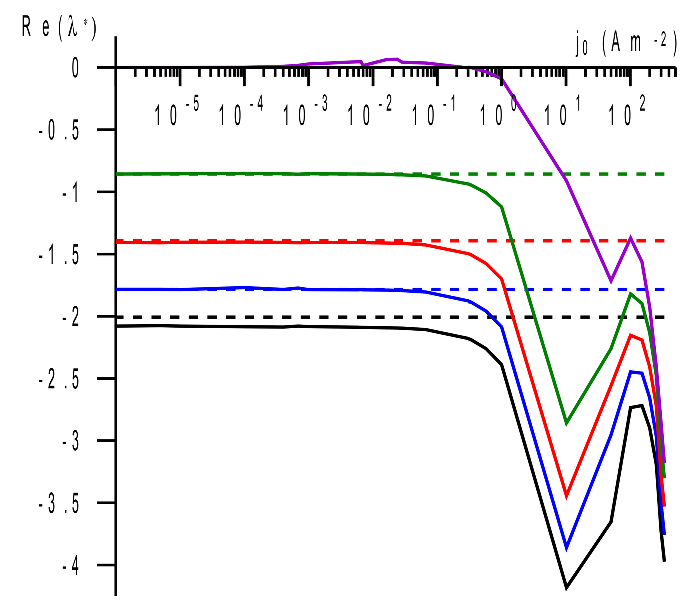
Blue: globally stable section
Green: section stable against a particular perturbation
Red: unstable section

- The increments that change sign are always real and the bifurcating perturbations are neutrally stable.
- There is a section of the high-voltage branch where the **first** non-fundamental mode **is stable => the possibility of observation of axially symmetric ring structures.**
- The second non-fundamental mode is always unstable.
- It is frequently assumed that **a discharge with a negative differential resistance is stable provided that the ballast is high enough** and the total resistance of the system discharge + ballast is positive.
- This assumption, while being valid at the high-current turning point, is violated at the low-current turning point. This is why (non-fundamental) modes constituting closed curves are possible.

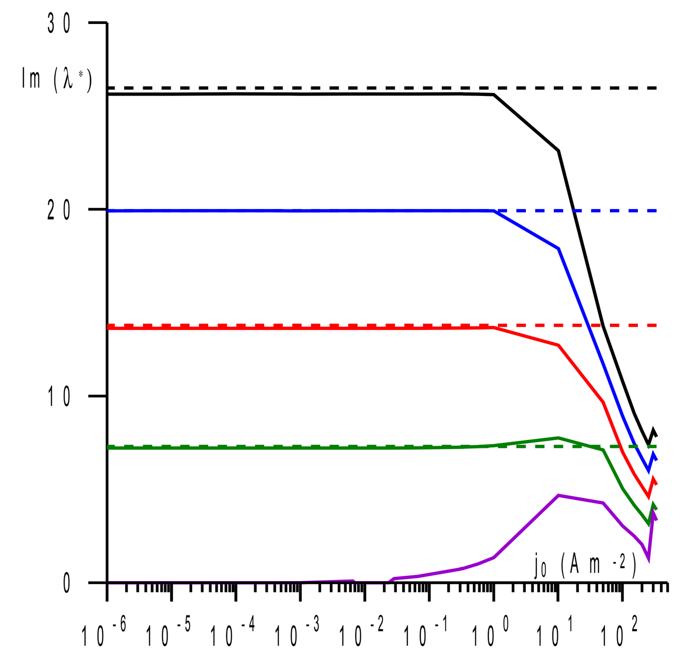
Analytical investigation of stability of Townsend discharge

- If diffusion, recombination, and effects of the order of μ_i/μ_e are neglected, then the normalized increment λ^* of 1D perturbations of 1D Townsend discharge may be found analytically:

$$e^{\alpha h} = \left(1 - \frac{\lambda^* - \alpha h}{\alpha h\gamma}\right)e^{\lambda^*}.$$



—— : Numerical results ----- : Analytical results



—— : Numerical results ----- : Analytical results

- Increments of all the modes of perturbation of the Townsend discharge except one do not change with current.
- These increments are describes well by the analytical theory.
- The Townsend discharge is stable against all the perturbation modes with a potential exception of one. Stability against the latter perturbation mode changes.
- How can this mode be described analytically?

Conclusions

- Increments of the perturbations can be both real and complex.
- The abnormal discharge is stable.
- There are windows of stability in the normal discharge => a possibility of observation of axially symmetric normal spots.
- There is a window of instability of the Townsend discharge and the loss of stability occurs through an oscillating perturbation. This conclusion conforms to observations and modeling of oscillatory behaviour of the Townsend discharge.
- There is a range of currents where the first non-fundamental mode is stable => a possibility of observations of steady-state axially symmetric ring patterns.
- The second non-fundamental mode is unstable.
- The assumption that a discharge with a negative differential resistance is stable provided that the ballast is high enough is not always true.

Acknowledgments

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